

UK Junior Mathematical Olympiad 2000

Organised by The United Kingdom Mathematics Trust

Tuesday 6th June 2000

RULES AND GUIDELINES : **READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING**

1. Time allowed: 2 hours.
2. **The use of calculators and measuring instruments is forbidden.**
3. All candidates must be in *School Year 8 or below* (English and Wales), *S2 or below* (Scotland), *School Year 9 or below* (Northern Ireland).
4. For questions in Section A *only the answer is required*. Enter each answer neatly in the relevant box on the Front Sheet. Do not hand in rough work.

For questions in Section B you must give *full written solutions*, including clear mathematical explanations as to why your method is correct.

Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Front Sheet on top.

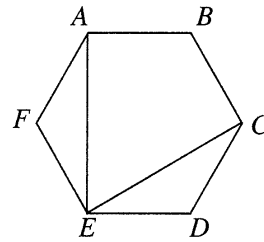
Do not hand in rough work.
5. Questions A 1-10 are relatively short questions. Try to complete Section A within the first hour so as to allow at least one hour for Section B.
6. Questions B1-B6 are longer questions requiring *full written solutions*. This means that each answer must be accompanied by clear explanations and proofs. Work in rough first, then set out your final solution with clear explanations of each step.
7. These problems are meant to be hard! Do not hurry. Try the earlier questions in each section first (they tend to be easier). Try to finish whole questions even if you can't do many. A good candidate will have done most of Section A and given solutions to at least two questions in Section B.
8. Numerical answers must be FULLY SIMPLIFIED, and EXACT using symbols like π , fractions, or square roots if appropriate, but NOT decimal approximations.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

Section A

A1 What is the value of $2000 + 1999 \times 2000$?

A2 $ABCDEF$ is a regular hexagon.
What fraction of the area of the hexagon is the area of the kite $ABCE$?

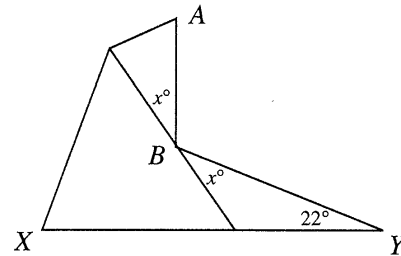


A3 Estimate, correct to the nearest mm, the side of a square of area 0.5 cm^2 .

A4 What is 20% of 30% of 40% of £50?

A5 The diagram shows part of a mosaic of tiles.
 AB is vertical and XY is horizontal.

What is the value of x ?



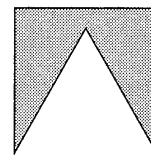
A6 The hour hand on the Mad Hatter's watch moves at the correct speed, but the minute hand moves one and a half times as fast as it should. Yesterday, it showed the correct time at 3 p.m. When did it next show the correct time?

A7 Six pupils have, between them, won three gold medals, two silver medals and a bronze medal in a painting competition. Unfortunately, their teacher, Mr. Turner, has lost all record of which medals should go to which pupils, so he allocates them by drawing names out of a hat. The first three names drawn receive the gold medals, the next two drawn have the silver medals and the bronze medal goes to the remaining pupil.

In how many different ways can the medals be allocated by this method?

A8 An equilateral triangle is cut out of a square of side 2 cm, as shown.

What area of the square remains?



A9 The machine which prints photographs at *Snippysnaps* runs for the same time every day. It prints colour photographs at a constant rate and monochrome (black and white) photographs at a different constant rate.

On Monday, the machine printed 2100 colour photographs and 2450 monochrome photographs.

On Tuesday it printed 2800 colour photographs and 1400 monochrome photographs.

On Wednesday, it printed only monochrome photographs.

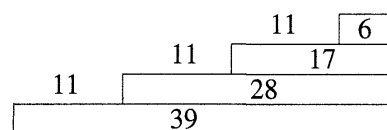
How many of these were there?

A10 It takes four gardeners four hours to dig four circular flower beds, each of diameter four metres. How long will it take six gardeners to dig six circular flower beds, each of diameter six metres?

Section B

Your solutions to Section B will have a major effect on the JMO results. Concentrate on one or two questions first and then **write out full solutions** (not just brief ‘answers’).

- B1** Kate has 90 identical building blocks. She uses all of the blocks to build this four-step ‘staircase’ in which each step, apart from the top one, is the same length.



- (i) Show that there are exactly two different ways in which it is possible to use all 90 blocks to build a six-step ‘staircase’.
- (ii) Explain fully why it is impossible to use all 90 blocks to build a seven-step ‘staircase’.

- B2** A crossnumber puzzle is like a crossword puzzle – except that the answers are numbers instead of words and each square contains one single digit. None of the answers starts with the digit 0.

How many solutions are there to this crossnumber?

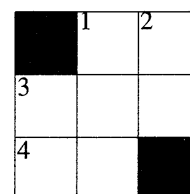
(You must use logic, not guesswork.)

Across

1. Square
3. Square
4. Square

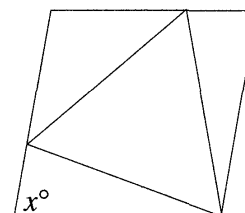
Down

1. Cube
2. Square
3. Cube times square



- B3** The diagram shows an equilateral triangle inside a rhombus. The sides of the rhombus are equal in length to the sides of the triangle.

What is the value of x ?



- B4** How many different solutions are there to the letter sum on the right?

Different letters stand for different digits, and no number begins with a zero.

$$\begin{array}{r} \text{J M C} \\ + \text{J M O} \\ \hline \text{S U M S} \end{array}$$

- (i) Explain why the sum of three consecutive integers is always divisible by 3.
- (ii) Is it true that the sum of four consecutive integers is always divisible by 4?
- (iii) For which k is it true that the sum of k consecutive integers is always divisible by k ?

- B6** X and Y play a game in which X starts by choosing a number, which must be either 1 or 2.

Y then adds either 1 or 2 and states the total of the two numbers chosen so far. X does likewise, adding either 1 or 2 and stating the total, and so on. The winner is first player to make the total reach (or exceed) 20.

- (i) Explain how X can always win.
- (ii) The game is now modified so that at each stage the number chosen must be 1 or 2 or 4. Which of X or Y can now always win and how?